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Linear Bulk Modulus Approximation for Sapphire

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If the pressure dependence of the bulk modulus is linear, ultrasonic measurements of the bulk modulus and the pressure derivative of the bulk modulus accomplished at pressures of a few kilobars can be used to predict compressions to pressures of hundreds of kilobars. The linear bulk modulus approximation for synthetic single-crystal Al_2O_3, sapphire, is examined by comparing extrapolations of ultrasonic data to shock compression data obtained in the range of 175 to 1500 kb. It is observed that shear strength effects influence the interpretation of the shock data to an extent at least as large as higher-order bulk modulus theories, and that shear strength corrections cannot be unequivocally applied to the shock data. When the shock data are corrected for a constant volume offset due to shear strength effects, the linear bulk modulus approximation is found to give an accurate fit to the shock data. Ultrasonic values for the pressure derivative of the bulk modulus are found to have a value about 15% higher than the modulus measured under shock conditions. This difference may be a consequence of changes in the properties of shock loaded sapphire induced by the large shear stresses accompanying the unusually large shear strength of sapphire.

Anderson [1966] has proposed that precise measurements of the bulk modulus and its pressure derivative determined at low pressures with ultrasonic techniques can provide the basis for accurately predicting compressions at very high pressures. This proposal is based on the assumption that the isothermal bulk modulus $B(P)$ is linear with pressure P; i.e.,

$$B(P) = B_0 + B_0'P \qquad (1)$$

where B_0 is the isothermal bulk modulus at atmospheric pressure and B_0' is the first pressure derivative of the bulk modulus. In testing this proposal Anderson obtained reasonable agreement between states extrapolated from ultrasonic data and states obtained in shock-compression experiments. General agreement with the linear bulk modulus approximation was obtained for a wide variety of solids ranging from compressible solids like NaCl to relatively incompressible solids like Al_2O_3. In spite of the general agreement, systematic differences were sometimes noted; in particular a difference of about 1 to 2% in relative volume was observed between ultrasonic and shock data for polycrystalline and single-crystal Al_2O_3.

Chung and Simmons [1968] compared more recent ultrasonic data [*Gieske and Barsch,* 1968] on synthetic single-crystal Al_2O_3 (sapphire) with shock data on sapphire (R. G. McQueen and S. P. Marsh as reported by *Anderson* [1966]; see also *van Thiel* [1967] and *Clark* [1966]) and found a difference of about 1% in relative volume between 500 and 1500 kb. To account for the discrepancy between extrapolated states based on ultrasonic data and the shock data, *Yu* [1968] proposed that the bulk modulus should include terms employing the second pressure derivative of the bulk modulus. In spite of the improved fit to the shock data with the quadratic bulk modulus approximation, it appears that other effects should be considered before the linear bulk modulus approximation is dismissed. In particular, these investigators neglected the effects of the shear strength of sapphire in comparing the shock data with hydrostatic data. Sapphire exhibits the largest shear strength observed for any solid [*Brooks and Graham,* 1966] and the shear strength can cause a systematic difference between the hydrostatic and shock response that may be large enough to account for the different compressions predicted by the linear bulk modulus and the quadratic bulk modulus theories.

As a part of an extensive study of oxides, *Ahrens et al.* [1969] improved the agreement between ultrasonic and shock data on Al_2O_3 by